

Carbon Cycle Lesson Plan for High School

Developed by: Maurice Smith, Monona Grove High School, Monona, WI

In collaboration with: Galen McKinley, University of Wisconsin – Madison

Copyright 2011/2012

High School Math/Science Carbon Cycle Applet lesson plans

These lesson plans are written in a modular format designed to accommodate varying amounts of class time dedicated to the study of climate change. If you have only one to devote to the subject, you will want to start with Day Eight. If you have two days available for study, you should consider using the plans for Days Seven and Eight. If you have eight days available, use days One through Eight in that order, and so on.

While these lesson plans are intended for use in a block-schedule setting of 90-minute class periods, they can easily be adapted to traditional 45-minute class periods by simply running each daily lesson over two class periods.

Below, the main topics covered in each of the eight days' lessons are summarized.

Day One: Collapse? The potential for global economic and environmental collapse in our lifetimes is a concern being voiced by more and more scientists and environmental activists in recent years. In his 2011 book World on the Edge, Lester Brown, a globally respected environmentalist and public policy advocate, makes it abundantly clear that the global economy—and human civilization itself—rely on “goods and services” provided free-of-charge by the natural world. Globally, however, these natural services are in steep decline worldwide due to human population growth, climate change and other factors. A detailed examination of Brown’s thesis offers an excellent introduction to a comprehensive climate change unit.

Day Two: Solutions! In order to force a hopeful focus on the unit, climate change solutions proposed in Brown’s World on the Edge, are explained in detail. High school students learning about these issues for the first time deserve (and appreciate) an unvarnished representation of the dangers we face, but they are poorly served if a “hope” piece is not present from the outset as well.

Day Three: Climate Change Basics. Atmospheric composition, the greenhouse effect, carbon sinks, natural global warming feedback loops and long-term climate variability, including the Milankovitch cycles and a detailed look at the Permian mass extinction, make up this science-based lesson.

Day Four: The History Lesson. Students beginning to understand the gravity global warming benefit greatly from learning the story of how human awareness of climate change emerged and blossomed over the past 150 years. Particular focus is given to the IPCC, the Denial Industry, the 2009 Copenhagen Conference and the prospects for global carbon emissions agreements.

Day Five: Basic Math Modeling. Students use their Algebra skills to building linear and exponential math models in order to predict future climactic states.

Day Six: Advanced Math Modeling. Students learn that areas under rate curves equate to total accumulated change in the given quantity. They apply this knowledge using the trapezoidal area formula and integrals to calculate areas under rate-of-CO₂-emissions

curves. The calculus-based math models created in this fashion offer a final method for predicting future climate states. This lesson requires no previous calculus training and all computations can be performed using the FNINT capabilities of a standard graphing calculator.

Day Seven: Pacala and Socolow's Wedges. In this lesson, students learn about the 2004 work of these two Princeton scientists who created a "menu" of options for reducing CO₂ emissions, each of which eliminates one detailed look at fifteen specific climate change solutions.

Day Eight: The Carbon Cycle Applet. Students use the Carbon Cycle Applet to predicting future climate states by testing assumptions about the carbon emission solutions we will employ in the future.

Carbon Cycle Background Material

Note: If you have not been using the full eight-day's worth of material, start the lesson with the following background information.

Background material:

- 1) Earth's average global temperature (AGT) has varied widely over its 4.5 billion year history. At times the AGT was so low the planet was a frozen ball of ice from pole to pole. At other times, the AGT was so high there was no year-round ice on the planet at all. It is a remarkable thing that only a few degrees change in AGT can make the difference between an ice age with two miles of ice covering Wisconsin, and an interglacial period like the one we have been enjoying for the past 10,000 years in which Wisconsin is ice free (except in winter). During ice ages like those we've been experiencing every 150,000 years or so in "recent" times, we see AGT fall to about 12 degrees C. During the warm interglacial periods, AGT rises to about 16 degrees. A seemingly minor change of only 4 degrees C in AGT makes a huge difference in global climate.
- 2) The periodic ice ages and warming trends we've been experiencing for the past couple of million years were all triggered by minor variations in Earth's orbit that occur on a predictable time scale. But very recently, starting about 200 years ago, a new factor has emerged that is having a huge effect on AGT—human CO₂ emissions and global deforestation. These human CO₂ emissions—which come from burning *Coal*, *Oil* and *Natural Gas* (CONG) has the capacity to raise AGT by up to six degrees C by the year 2100. If that happens we will not recognize Earth's climate any longer—it will be like living on a new planet with much harsher weather, more deserts, less land for growing food, new ranges for many diseases, as much as 50% fewer species, and far less ability to support human civilization.
- 3) The reason burning CONG (together, known as fossil fuels) impacts AGT is because of carbon. All fossil fuels are energy-rich collections of carbon and hydrogen which release huge amounts of energy when burned. One barrel of oil, for example, contains as much energy as the amount used by 25,000 hours of human labor—and we are currently burning more than 1000 barrels of oil per second. That means oil provides modern civilization with 25 million human-hours worth of energy every second. That means CONG is like buried treasure—but it comes with a price. When we burn fossil fuels, the carbon separates from the hydrogen and binds with oxygen in the atmosphere to produce carbon dioxide—CO₂. A little CO₂ in the atmosphere is a very good thing—because of the way it traps heat leaving the Earth's surface, the small amounts of CO₂ in our atmosphere are responsible for keeping Earth's surface from freezing solid.
- 4) During the past million years or so, CO₂ levels have ranged from about 180

parts per million (PPM) during ice ages to 280 PPM during warm interglacial periods like the one we are in now. In percentage terms, the atmosphere is 78% Nitrogen and 21% oxygen, leaving just 1% left for all of the other gasses. CO₂ at 280 PPM is only 28 one thousandths of one percent of the atmosphere—but the atmosphere is so delicately balanced that the tiny change from 180 to 280 PPM is the difference between 12 degrees C (ice age) and 16 degrees C (present day). Humans—even seven billion of us working together—don't seem big enough to change the balance of the atmosphere, but we have managed to do just that. Since we started burning fossil fuels a few hundred years ago we have raised the CO₂ level in the atmosphere from 280 to 392, setting in motion the melting of all things frozen, causing sea levels to begin rising and altering the weather by allowing the atmosphere to hold an additional 5% more water vapor. The increase from 280 to 392 PPM atmospheric CO₂ (ACO₂) has raised AGT by about 0.5 degrees C so far with another 0.8 degrees C temperature increase on the way. If we continue to burn fossil fuels at the current rate, we will reach 450 PPM ACO₂ within 30 years and guarantee a AGT increase of two degrees C compared to “normal.

- 5) Again, that 2 degree C average global temperature (AGT) increase doesn't sound like much, but scientists believe it would be enough to cause runaway global warming from natural carbon sources. For example, huge amounts of carbon are stored as methane (CH₄—the same as natural gas) in the frozen tundra and underwater off the coasts of earth's most northerly land masses. If these methane deposits are released as tundra thaws in the hotter climate humans are creating, AGT could rise far more than two degrees C. The worse global warming gets, the more the tundra will melt, and the more the tundra melts, the more carbon it will release. This in turn leads to still higher AGT and the cycle of hotter-and-hotter continues. Once these natural feedback loops kick in, they are impossible to stop, so it's important that humanity slows its CO₂ emissions rate and avoids the 450 PPM ACO₂ figure.
- 6) Human Civilization puts about 9 Gigatons (GT) of carbon into the atmosphere every year. One Gigaton is one billion metric tons, or 2.2 trillion pounds. (By comparison, the weight of all humanity is about one trillion pounds). One Gigaton is also the same as one trillion grams—also known as one Petagram (PG). Remember—one GT is the same as one PG. When those 9 GT of carbon combine with oxygen in the atmosphere, the CO₂ produced weigh about 33 GT, so don't get confused when you hear that humans emit 9GT of carbon and 33GT of CO₂ per year—they both mean the same thing. For our purposes, it's easier to just talk about carbon emissions for a moment, so we'll be dealing with GT carbon from here on out. What happens to all that carbon and the CO₂ it creates? About 40% of it is absorbed by the oceans, another 30% is taken up by land plants and the remaining 30% enters the atmosphere where it contributes to global warming. We are mighty lucky the oceans and land masses absorb so much of our carbon. These “carbon sinks” keep the atmosphere from absorbing as much CO₂ as it would otherwise.

- 7) Unfortunately, the ocean and land sinks are becoming less able to absorb our excess carbon. Ocean sinks appear to be weakening because warmer ocean water absorbs less CO₂ than cold water does, and land sinks may be declining due to the growth of the earth's deserts. IN both cases, the worse global warming gets, the less effective our carbon sinks seem to work. If human carbon emissions continue at the current rate, these natural carbon sinks could continue declining, which would allow more human carbon emissions to collect in the atmosphere, further raising AGT and leading to another natural feedback loop in which higher AGT leads to still higher AGT. Again, scientists believe 450 PPM ACO₂ is a cutoff point beyond which these feedback loops become unstoppable.

- 8) Because of population growth and increases in the standards of living for many of the world's poor people, the 9 GT of Carbon humans now emit annually is expected to rise to 16 GT C per year by 2060. However, we know that by 2060 we know we must reduce global carbon emissions to 4 GT per year—the amount our natural sinks can absorb. This means we need to cut 12 GT C (16 minus 4) from our global carbon diet in the next 40 years or so to avoid runaway global warming from natural sources and the destruction of our natural sinks. There is no single, enormous change we can make would reduce our carbon emissions by 12 GT per year. But many smaller strategies exist that would cut one GT C per year. Two scientists from Princeton University, Stephen Pacala and Robert Socolow, proposed fifteen strategies back in 2004 each of which would cut one GT of carbon from the annual global emissions total. The triangle-shaped pieces removed from the total carbon budget resemble wedges, which is why we call these fifteen strategies Pacala and Socolow's "wedges". The following website explains each of the wedges: <http://cmi.princeton.edu/wedges/> then choose "introduction." (Take students to this website or distribute a handout based on the information handout located there and discuss each of the wedges in order to familiarize students with the concepts involved.)

This marks the end of the background information. Students should now be able to follow the directions below for the following activities.

Day Five: Basic Mathematical Modeling

At the conclusion of this lesson students will:

KNOW:

The slope intercept form of the linear equation ($y = mx+b$);

The general form of the exponential equation ($y = bg^x$)

UNDERSTAND:

- 1) The limitations of using linear and exponential equations when modeling real world phenomena.
- 2) That continued CO₂ emissions at current rates leads to predictable outcomes for atmospheric carbon (AC) levels and average global temperatures (AGT).

BE ABLE TO DO:

- Write linear equations from real-world situations and use them to predict future states of the given system.
- Write exponential equations from real-world situations and use them to predict future states of the given system.
- Refine mathematical models by selecting appropriate data points.

Introduction: Building Mathematical Models

One of the great things about math is that it can allow us to predict the future based on assumptions we make about the past. When we build equations to help us make those predictions, we are building mathematical models. This lesson on building simple linear and exponential math models would be appropriate for students from 6th grade onward who have some grasp of variables and simple equation solving. Much of the discussion in stages two and three could be skipped for students with typical first-year algebra proficiency.

Stage One: Stating the question.

If humans continue releasing CO₂ at current levels by burning coal, oil and natural gas, what future AC and AGT levels can we expect?

It's pretty tough to try and answer that question accurately, because there are so many variables. Land use changes (like clearing forests to plant crops) cause additional CO₂ releases and need to be factored in. The land and ocean carbon sinks that absorb CO₂ before it reaches the atmosphere are in flux and need to be considered. The role of concrete production (which accounts for billions of tons of CO₂ emissions every year) requires our attention. Possible new natural sources of CO₂ emissions (from thawing tundra, for instance) represent another factor at work that should be examined.

However, in answering this question we will simplify our work a great deal and focus strictly on CO₂ emissions from burning coal, oil and natural gas (CONG). We lose accuracy when we ignore those other factors, but that's the price we have to pay for the ability to build a math model we can understand.

Stage Two: Understanding Linear Equations

The equation $y = mx + b$ is one of the most important in all of algebra. It's called the linear equation because the graph of it's (x, y) ordered pair solutions is a line.

In the linear equation, **y stands for the “stuff we're counting”** in the problem situation (rocks, rabbits, PPM of CO₂ in the atmosphere), **x usually stands for time units**, and **b represents the starting amount of “stuff”** (or “stuff at time zero”). B is also known as the y-intercept of the equation.

M is the most important variable in the equation—it stands for the rate at which y changes as x increases. How many rocks per week are you adding to your rock collection? How many rabbits per month are being born on your rabbit farm. How many additional GT of C are entering the atmosphere every year? These are all m values (rates of change of y over x).

Here's another way of viewing **M**—it tells you how much y goes up if x goes up by one. **M** is also called the “slope” of the equation because it tells you how steep the line is when you graph the solutions.

Since the equation $y = mx + b$ contains both the slope (m) and the y intercept (b) it is sometimes called the slope-intercept equation of the line.

Here's how the slope intercept equation works. Let's say you collect rocks and add the same amount to your collection every week. You start with 10 rocks in your collection (**that means b is 10.**) Let's also say you add 7 rocks to your collection every week (**this means m is 7**). Substituting these values into $y = mx + b$, we find the linear equation for your rock collection population is:

$$y = 7x + 10.$$

You can use this equation to predict how many rocks you'll have in the future. How many rocks wil you have after 20 weeks? Simply substitute 20 for the time variable (x) and simplify:

$$\begin{aligned}y &= 7x + 10 \\y &= 7(20) + 10 \\y &= 140 + 10 \\y &= 150.\end{aligned}$$

Start your the linear equation and substitute 20 for x
Simplify...
Simplify..
You would have 150 rocks after 20 years.

Stage Three: Writing Linear Equations

Sometimes we are not sure about our starting value or our rate of increase and we have to use the slope formula to find the m value in our equation. The slope formula looks like this:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{where the } x_1, x_2, y_1, \text{ and } y_2 \text{ variables just stand for any two ordered pairs from the data you collected.}$$

Here's a sample problem situation to help make things clearer:

Suppose you started collecting rocks a while back and kept track of your collection. Your notes indicate you added the same amount of rocks to your collection every week, which means your collection was growing in a linear fashion. *You have to be sure the same amount is adding every time unit--otherwise you can't use a linear math model!!!*

Now that we know we have a linear situation, we need to define our variables. Simple: Let x stand for weeks since starting the rock collection and let y equal number of rocks at any given time.

Next, we need to find two (x, y) ordered pairs. Let's imagine your rock collection data says you had 700 rocks after 6 weeks and you had 790 rocks after 9 weeks. *This means the two ordered pairs you have to work with are: (6, 700) and (9, 790).*

Now we find the slope (m) value for the linear equation. In these ordered pairs, the first numbers (6 and 9) are the x values and the second numbers (700 and 790) are the y values. In order to keep them straight, we can call the x value 6 " x_1 " (pronounced x-sub-one) and we can call the x value 9 " x_2 " (x-sub-two). Likewise we can let y_1 stand for the y value 700 and y_2 can represent the other y value, 790.

In this way, our two ordered pairs now look like this:

(6, 700) is our (x_1, y_1) and (9, 790) is our (x_2, y_2)

And now that we know the meaning of the variables, we can plug numbers into the slope formula from the previous page:

$$m = \frac{y_2 - y_1}{x_2 - x_1} \quad \text{can now be re-written as ...}$$

$$m = \frac{790 - 700}{9 - 6} \quad \text{which can be simplified to...}$$

$$m = 90 / 3 \quad \text{which reduces to...}$$

$$m = 30.$$

The slope value (**m**) **turns out to be 30 rocks per week.** What this means is, if you had 700 rocks after 6 weeks and 790 rocks after 9 weeks, *you must have been adding rocks to your collection at the rate of 30 rocks per week.* And, of course, this rate of 30 rocks per week means the m value in your linear equation is 30.

The $y = mx + b$ linear equation for your rock collection can therefore be rewritten as:

$$Y = 30x + b.$$

D) All we have to do now is find the b value--the starting amount of rocks in your collection. This you can easily do by simply choosing either ordered pair from your data and substituting them in for x and y in the linear equation you are building. It doesn't matter whether we use the (6, 700) or the (9, 790) data points--I'll choose the first one. Here's what the next step looks like:

$$Y = 30x + b$$

Let $x = 6$ and $y = 700$

$$790 = 30(6) + b$$

Now, solve this equation for b--simplify

$$790 = 180 + b$$

then add -180 to both sides

$$790 + -180 = 180 + b + -180$$

simplify further--

$$610 = b$$

And that means the b value is 610.

Finally, we rewrite the linear equation one more time with $m = 30$ and $b = 610$ to get:

$$Y = 30x + 610.$$

E) This equation describes your rock collection's population--and we were able to find the equation starting with just two ordered pairs. As usual, we can now use the equation to answer questions about the future of your rock collection. How many rocks will you have after 100 weeks of collecting? Just substitute 100 for x and simplify:

$$Y = 30x + 610$$

$$Y = 30(100) + 610$$

$$Y = 3000 + 610$$

Y = 3610 rocks will be in your collection after you have collected for 100 weeks.

Stage Four: Finding a linear equation for atmospheric carbon.

We know the atmosphere held about 280 parts per million (PPM) of CO₂ back in the year

1800. We also know the atmosphere contained about 388 PPM CO₂ in the year 2010. If atmospheric carbon levels continue to increase at the same rate they did for the last two hundred years, what will the PPM value be in 50 years? We can use our linear equation skills to find out.

A) First, we need to *name what our x and y variables stand for*: x will represent time (in years) and y will represent PPM CO₂ in the atmosphere.

B) Next we need to *find two ordered pairs*. We could choose (1980, 280) and (2010, 388), for our two ordered pairs--but that would mean x was measuring “years since year zero,” and we would be forced to deal with really large x values like “1980” and “2010”. Also, we would end up with a strange b value if we let x be “years since 2011 years ago.”

So, instead, we would find it but easier to create our linear equation if we *let x = 0 stand for right now*--the year 2011. If we decide to let “now” stand for “time zero”, then the x value for 2010 (last year) would be x = -1 (for one year ago). And the x value for the year 1800 would be x = -211 (because the year 1800 was 211 years ago).

Following this method, our two ordered pairs would be:

(-1, 388) meaning one year ago (2010) there were 388 PPM CO₂ in the atmosphere, and:

(-211, 280) meaning in the year 1800 (211 years ago) there were 280 PPM of Co₂

C) Next, we *use the slope formula to find our m value*:

$m = \frac{y_2 - y_1}{x_2 - x_1}$ The slope formula can be re-written as...

$m = \frac{388 - 280}{-1 - -211}$ which can be simplified to...

$m = 108 / 210$ which rounds to...

$m = .51$ when we divide.

This means CO₂ levels in the atmosphere (the y values) are increasing by about .52 PPM per year. Our linear equation now looks like this:

$$Y = .51x + b$$

Next we find the b value by letting either ordered pair from step B stand in for x and y. I'll choose the (-1, 388) ordered pair--

$$Y = .51x + b \quad \text{let } x = -1 \text{ and } y = 388$$

$$388 = .51(-1) + b \quad \text{simplify--}$$

$$388 = -.51 + b \quad \text{Add the opposite of } -.51 \text{ to both sides--.}$$

$$388 + .51 = -.51 + b + .51 \quad \text{simplify--}$$

$$388.51 = b$$

And now we have our b value--388.51--which means we expect 388.51 PPM CO₂ in the atmosphere in “year zero”, which is this year.

Our linear equation now looks like this:

$$Y = .51x + 388.51.$$

Now we can use the equation to predict levels of CO₂ in the atmosphere in fifty years. Just let $x = 50$ and simplify--

$$Y = .51x + 388.51 \quad \text{Let } x = 50\text{--}$$

$$Y = .51(50) + 388.52 \quad \text{simplify--}$$

$$Y = 25.5 + 388.52 \quad \text{simplify--}$$

$$Y = 414.02$$

This means that fifty years from now if atmospheric carbon dioxide levels increase at the same rate we will see 414.52 PPM of CO₂ in the atmosphere fifty years from now.

Stage 5--Refining the model.

Our first math model for predicting future ACO₂ levels developed above in Stage 4 can be improved upon if we *refine our assumptions*. In that first model we used data from 1800 to create our slope. What might happen if we used more recent data? Try it yourself. Using the same process from the Stage 4 example, predict the future ACO₂ level in PPM using these two facts:

In 1960, there were about 315 PPM CO₂ in the atmosphere and by 2010 that figure had risen to about 388 PPM. Show all your work on your own paper, of course.

If your work resulted in the new linear math model $y = 1.46x + 339.46$, good for you. If not, ask for help now.

What does this new model say about ACO₂ levels 50 years from now? You should get

462.46 PPM CO₂.

*Let's next explore why we got a different slope and y intercept in our second equation. Obviously, it's because we used a different ordered pair based on a different data point--but why did **m** get larger? Think about that before you read further...*

Hopefully you see that **m** got larger because the rate at which AC is entering the atmosphere has not stayed constant over the last 200 years. Years ago AC levels were creeping up slowly. More recently they have been rising much faster. When we used more recent AC data, we created a more accurate estimate, and the new 462 PPM value is probably closer to reality than our old 414 PPM estimate for PPM AC in fifty years.

Stage 6--Linear Model Number Three

Let's build another linear model for future ACO₂ in PPM based on the best and most recent data we have. Namely, over the past two years the atmosphere has added an average of 2.2 PPM per year (so that's our **m** value in PPM CO₂ per year) and we are currently at about 390.31 PPM CO₂ (so that's our **b** value). ([Data from http://co2now.org/](http://co2now.org/))

Build a new linear model with that data--and if you're not sure how, look back up at Stage Two for a minute. Show all work on your own paper, as usual.

Did you come up with $y = 2.2x + 390.31$? If so, excellent. If not, ask for help now.

Let's use the new model to predict AC levels fifty years from now. Did you get 501.31 PPM? Good work. Since this model uses the "freshest" data, it's probably the most accurate linear model we can create.

Stage Seven—A New Use for the Linear Model

You may recall now that at 450 PPM CO₂, scientists predict some of those natural feedback loops might kick in. Lets use this linear math model in a new way to predict how many years until we hit 450 PPM ACO₂. This time, substitute 450 in for **y** in the equation and solve for **x**.

$$Y = 2.2x + 390.31$$

Start with the linear equation

$$450 = 2.2x + 390.31$$

Substitute 450 in for y

$$450 + -390.31 = 2.2x + 390.31 - 390.31$$

Add -390.31 to both sides

$$59.69 = 2.2x$$

Simplify--

$$59.69/2.2 = 2.2x/2.2$$

Divide both sides by 2.2 and simplify

27 = x Looks like in 27 years at current rates we would hit 450 PPM of atmospheric carbon.

Stage Eight--Linear Model Number Four

In this model, let's explore the connection between AC and AGT. *Instead of looking at how AC changes compared to time, we'll see how AGT changes compared to AC levels.*

We know AGT was about 13.5 degrees C 20,000 years ago (during the last ice age) when AC levels were at about 180 PPM. We also know that AGT rose to about 15.5 degrees C 10,000 years ago (when the ice age ended) and CO₂ levels hit around 280 PPM then. Finally, we know that today AGT is about 16 degrees C and AC levels are hovering around 390 PPM. From this information we ought to be able to get three data points--but which measurement do we use for x? AGT or AC level? Here's how to figure that out--

We always call Y the "dependent variable" and x the "independent variable." So ask yourself--does the AC level depend on AGT, or does AGT depend on AC? Well, since AC *causes* the rise in AGT, we say that AGT depends on AC. Since AGT is "doing the depending," we call it the dependent variable--so we represent it with Y. The AC level, on the other hand, is independent--and we use X to represent it.

With this in mind, every (x, y) ordered pair will stand for an (AC, AGT) data pair. Here are the three data points to work with:

Data Point 1: (180, 13.5) from 20,000 years ago

Data Point 2: (280, 15.5) from 10,000 years ago

Data Point 3: (390, 16) from today's AC and AGT values.

Take a moment now and find three different linear models using two different data points each time. Use each one to estimate the AGT value in degrees C that would result from 450 PPM of AC.

Stage Nine--Exponential Equation Basics.

Now that you're an expert in creating linear models to make predictions about future AC levels and AGT, let's add on to your skills by exploring a more accurate kind of math modeling equation. Back up in Stage 2 it was mentioned that : *"You have to be sure the same amount is adding every time unit--otherwise you can't use a linear math model!!!"*

The thing is, we know we have not been adding equal amounts to the AC PPM level every year. Instead, the amount being added is going up every year. When the rate is not constant you can't use a linear model to make very accurate predictions about the future--but you can use exponential models instead.

The exponential equation looks like this: $y = bg^x$ and it works for situations where the rate of change of y is itself changing. Here's what the variables stand for:

Y is still the "stuff" being counted;

X is still time units;

B is still the "starting amount of stuff";

G stands for the growth factor--sort of like the slope in a linear equation, it is by far the most important variable in the equation.

G can be further broken down by this formula: $g = 1 + r$ where r is the percentage rate of change in Y from one time unit to the next.

Here's an example to make things clearer:

Suppose you start a rabbit farm with 50 bunnies and your rabbit population increases by 23% per month. This would mean $b = 50$ (the starting amount) and $r = 23\%$. When we convert r from a percentage to a decimal we find that $r = .23$. And since $g = 1 + r$, our g value must be $1 + .23$ or $g = 1.23$.

Now we can substitute 50 for b and 1.23 for g in the exponential equation:

$Y = bg^x$ now becomes

$$Y = 50(1.23)^x$$

How many rabbits will we have in 50 months? *Let $x = 50$ and evaluate.* And don't be shocked if you came up with a number in the millions--exponential growth can make y values really huge really fast.

Now let's apply this new exponential equation model to global warming. Last year the atmosphere contained about 388 PPM of CO₂. This year we're up to 390.31 PPM.

A) We can easily *find the rate of increase* (the r value) by first finding out how much AC went up:

$$390.31 - 388 = 2.31 \text{ PPM} \quad \text{This is the increase from last year to this year in PPM of AC}$$

B) Next divide the increase amount (2.31 PPM) by last year's amount (388PPM):

$$2.31/388 = .0059 \quad \text{This means the } r \text{ value is } 0.0059$$

C) We can now add this r value to 1 to find g :

$$g = 1 + r$$

$$g = 1 + 0.0059$$

$$g = 1.0059$$

D) Our starting amount could be 390.31 if we let x measure years from now (2011) so that's our b value ($b = 390.31$).

E) Now when we substitute them in to $Y = bg^x$ we get:

$$Y = 390.31(1.0059)^x$$

And now, as always, let's predict 50 years from now what the PPM AC levels might be. Let $x = 50$ and simplify. Show your work--

Did you come up with 523 PPM? Nice job. If not, ask for help now.

Finally, back in Stage 6 we used our best linear model to estimate how many years until 450 PPM AC--and we came up with 27 years. What does our exponential model have to say about the year we hit 450? This time we have to let y equal 450 and guess and check x values until the right hand side of the equation gets close to 450. (There is a way to solve the equation without guessing and checking. It involves a math object called a logarithm. For extra credit check out this website and practice solving our exponential equation exactly using logarithms. <http://www.purplemath.com/modules/solvexpo2.htm>

$$Y = 390.31(1.0059)^x$$

Start with the original equation

$$450 = 390.31(1.0059)^x$$

Substitute 450 in for x .

$$450/390.31 = 390.31(1.0059)^x/390.31$$

Divide both sides by 50

$$1.15 = 1.0059^x$$

Now guess and check to find an x that works--show your work.

Did you come with about 23 years? Excellent. If not, ask for help.

Comparing this to our linear equation prediction from stage 6 we see a slightly shorter amount of time required for us to reach 450 PPM when the ever-increasing rate of AC Level growth is taken into consideration.

Stage Ten--Building Exponential Models

Sometimes we need to work a little harder to get our exponential models. Here's a look at the steps we follow if we are not "given" a starting amount and g value to start with.

Let's build an exponential model for human Carbon emissions compared to time. In 1980,

humans were releasing about 4.9 GT of C annually from burning CONG. By the year 2000 that figure had risen to 6.3 GT per year. If we assume human C emissions (y) depend on the year (x), and if we assume x measures “years from now” our (x, y) ordered pairs look like this:

(-31, 4.9) 31 years ago in 1980 we released 4.9 GT C per year.
(-11, 6.3) 11 years ago in 2000 we released 6.3 GT C per year.

Here’s how to use these two data points to find our exponential model:

A) Substitute one ordered pair into the equation for x and y:

$$4.9 = bg^{-31}$$

B) Substitute the other ordered pair in for x and y.

$$6.3 = bg^{-11}$$

C) Now divide the two equations:

$$\begin{array}{l} 4.9 = bg^{-31} \\ \text{divided by } 6.3 = bg^{-11} \end{array} \quad \text{looks like this:}$$

First, calculate 4.9 divided by 6.3--round it to 0.78

Second, calculate b divided by b--that would be just plain old one

Third, calculate g^{-31} divided by g^{-11} . To do this, subtract exponents.

$$-31 - -11 = -20 \quad \text{--and the result becomes } g^{-20}.$$

Put the three results together and the new equation is:

$$0.78 = 1 * g^{-20}$$

Notice, this equation has lost it’s b value because of step 2 above. Also notice that we can multiply the 1 by the g term to get:

$$0.78 = g^{-20}.$$

D) Next, we solve the equation for g by raising both sides to the reciprocal power. The reciprocal of -20 is -1/20--so that’s the exponent we use:

$$0.78^{(-1/20)} = (g^{-20})^{(-1/20)}$$

On the right side of the equation, the exponents multiply leaving us g^1 which is, of course,

just **g**. On the left side, we *calculate* $0.78^{(-1/20)}$ to be **1.0125**. Our equation now looks like this:

$$1.0125 = g$$

This means we have found our **g** value for the exponential equation.

E) Next we rewrite the exponential equation with 1.0125 in place of **g**:

$$Y = b(1.0125)^x$$

Our last job is to find the **b** value--and all we have to do is pick wither original ordered pair and substitute in for **x** and **y**. (I'll choose the first ordered pair, but we would get the same answer regardless which one we used.)

$$4.9 = b(1.0125)^{-31} \quad \text{Simplify--}$$

$$4.9 = b(0.68) \quad \text{Divide both sides by .0017}$$

$$4.9/0.68 = b(0.68)/0.68 \quad \text{Simplify--}$$

$$7.205 = b \quad \text{and we have our } b \text{ value.}$$

Put the **b** and **g** values together in the exponential equation and we have:

$$Y = 7.205(1.0125)^x \text{ as our model for annual human carbon emissions.}$$

Stage Eleven--An Exponential Model for future AC Levels

Using the skills you developed in Stage Ten above, create an exponential model for future AC levels in PPM using the following two data points: In 1980 the PPM level was 350 PPM and it increased to 370 PPM by the year 2000. Show all work.

Did you get this equation: $y = 381.44(1.0028)^x$ (or something pretty close)? If so, great. If not, ask for help.

Stage Twelve--Practice Problems

Two of the big GW questions we can ask and answer using simple high-school math are: How much AC can we add to the atmosphere before we risk setting off the NFL's. (We already know the answer to this one--it's about 450 PPM).

How long will it take us to reach that AC level if we continue releasing CO₂ by burning CONG at current rates? (The answer here varies based on your assumptions--20 to 40 years is the current most common range scientists talk about).

We learned how to build and use linear and exponential math models to answer these questions in stages Two through Eleven above. Here is a fictional scenario that will help you practice these math skills.

Welcome to the fictitious planet, Bob, orbiting its happy little sun in some far-off galaxy. Humans exist on Bob, too, and they found buried CONG there just like we did here on earth. Upon finding that buried energy treasure, they did what we did (or anyone else would have done)--they lived it up! They mined all the CONG they could get their hands on and burned it up during their great "industrial revolution" period. And just like us, they soon discovered that there was a downside to the buried treasure--the genie in the bottle turned out to be toxic because of the CO₂ emissions associated with burning all that CONG. Bobian researchers predicted Bobian NFL's would kick in at 30 Degrees C.

Here's the data from Planet Bob

Year	AGT	PPM ACO ₂	GT AC
2700	20 degrees C	400 ppm	
2900 (now)	26 degrees C	1600 ppm	
?	30 degrees C		

- 1) Create a linear model for AGT in terms of ACO₂ and use it to estimate PPM ACO₂ level that produces AGT of 30 degrees.
- 2) Create a linear model for PPM ACO₂ in terms of years and use it to estimate year we hit that PPM level.
- 3) New data comes to light that in the year 2880, the PPM level of AC was 1200. This means the PPM level is rising faster now than it was in the past, so we ought not to use a linear model to predict future atmospheric states. Build an exponential model for AC in terms of years and use it to predict what year planet Bob reaches the ACO₂ PPM level from question 1.

Day Six: Advanced Mathematical Modeling

**Day Seven: Pacala and Socolow's Wedges:
An Approach to Reducing Human CO₂ Emissions**

1) Explain to students that, “the race is on. In Lane One, it’s humanity’s ever-accelerating annual carbon dioxide emissions. In Lane Two, we have nature’s decreasing ability to absorb the CO₂ we release. And in lane three, we have the natural feedback loops getting ever-closer to kicking in. The current amount of carbon dioxide in earth’s atmosphere (atmospheric CO₂ or ACO₂) is causing the planet’s average global temperature (AGT) to increase. If ACO₂ levels rise much beyond the current 395 parts per million (PPM) value, natural stores of carbon long frozen safely away could be released, adding enormous new quantities of ACO₂ and driving AGT far higher than the earth has seen for the past 55 million years. This would spell disaster for the global economy, human civilization and millions of species. The natural sources of carbon and other natural global warming feedback loops simply must not be allowed to “kick in.” If they do, we will no longer be able to exert any control over the increase in ACO₂.

Here’s the good news: At the moment we have two things going for us in this great race—first, we can control how much CO₂ we release every year from burning fossil fuels. It is definitely within our power to reduce our CO₂ emissions. Second, nature is currently still absorbing some of the CO₂ we produce—about 4GT gigatons (GT) worth of Carbon (C) every year, in fact. The bad news is Humans are currently producing about 9 of carbon (C) every year by burning fossil fuels and altering land use patterns. (Recall—one GT is also known as one Petagram, or PG). Because of population growth and increases in the standards of living for many of the world’s poor people, that value is expected to rise to 16 GT C per year by 2060. However, we know that we can’t allow that increase to occur—if we do, the natural feedback loops will certainly kick in.”

2) Go to <http://cmi.princeton.edu/wedges/> then choose “introduction.” Post a sketch of the annual CO₂ emissions rate graph from 2000 to 2060, highlighting the triangle with vertices at (2010, 8GT/y), (2060, 16GT/y) and (2060, 4GT/y). Use the graph to illuminate the following explanation:

“Therefore, by 2060 we know we must reduce global carbon emissions from the projected 16 GT/year all the way down to 4 GT per year—the amount our natural sinks can absorb. This means we need to cut 12 GT C from our annual global carbon diet in the next 40 years or so to avoid runaway global warming from natural sources. There is no single change we can make that would reduce our carbon emissions by 12 GT per year. But many strategies exist that would each cut *one* GT C per year. For example, if we simply increased fuel efficiency on the world’s auto fleet to 60 MPG average, we would cut one of the 12 GT of annual CO₂ emissions we need to eliminate.”

3) Sketch a one-GT wedge onto the graph. Label it “Savings from increased auto efficiency.” Continue with the explanation:

“Two scientists from Princeton University, Stephen Pacala and Robert Socolow, proposed fifteen strategies back in 2004 each of which would cut one GT of carbon from the annual global emissions total. The triangle-shaped pieces removed from the total carbon budget resemble wedges, which is why we call these fifteen strategies Pacala and Socolow’s “wedges”.

The following website explains each of the wedges: <http://cmi.princeton.edu/wedges/>. Take students to this website or distribute a handout based on the information handout located there and briefly discuss each of the wedges in order to familiarize students with

the concepts involved.

4) Break the students up into five groups. The groups:

- A) Efficiency and Conversion strategies; (Wedges 1-4)
- B) Carbon Capture and Storage strategies: (Wedges 5-7)
- C) Coal Replacement strategies: (Wedges 8-9)
- D) Renewable Energy strategies: (Wedges 10-12)
- E) Biostorage Strategies. (Wedges 13-15)

Each group will be assigned from two to four wedges and asked to produce the following to share with the rest of the class:

- 1) A poster explaining your category and showing the wedges contained in your category. This poster must contain a short descriptive title of five to ten words in length that paraphrase or re-define your category, a short synopsis of the wedges you are assigned and a graphic organizer that helps students from other groups visualize the category and wedges through words and images.
- 2) A poster for each wedge employing words and images that helps students from other groups understand the concept of the wedge.
- 3) Your answer to the question—if you had to choose one of these wedges to abandon, which one would it be and why?

After each group has completed their posters and decided which wedge to abandon, the groups present their posters to the whole group.

Next, class votes on which two of the five abandoned wedges to keep. The other three are then deemed the three least realistic wedges and the other 12 that “make the cut” are the most promising of the CO₂ emissions reductions strategies.

(An alternate lesson plan employing the wedges can be found under the “The Wedges Game” tab on the CMI website).

Day 8. The Carbon Cycle Applet Project:
Pacala and Socolow's Wedges and the quest for a below-450 solution.

STUDENT ACTIVITY INSTRUCTIONS

In this activity it will be your job to create a plan for solving global warming. You will select twelve of Pacala and Socolow's wedges, decide what year each wedge will be implemented, then enter your solution plan into the Carbon Cycle Applet and see if your plan has managed to keep ACO₂ levels below 450 PPM.

Directions:

- 1) The plan you create needs to include 12 of the 15 wedges proposed by Pacala and Socolow. If humanity employs 12 wedges by 2040 we will have reduced our global annual carbon emissions from the projected 16 GT per year to 4 GT per year—the amount natural sinks and sources can absorb for us without allowing more to build up in the atmosphere).
- 2) For each wedge you choose, explain why you think it is realistic that the wedge could be implemented and describe how the wedge will reduce global carbon emissions by 1 GT—in other words, what mechanism's are at work that lead to the carbon reduction from each wedge you choose?
- 3) For each wedge you choose, you must state when it will be placed into action. The only available years to choose are 2020 and 2040.
- 4) After your have created your plan, run the Carbon Cycle Applet and record the following:
 - A) The fossil fuel data you input for 2020, 2040, 2060, 2080 and 2100;
 - B) Maximum CO₂ PPM reached during the projection run;
 - C) Ending value of CO₂ reached by the projection run.
 - D) A sketch of the shape of the projection curve, labeling max point and end point.
- 5) Two important reminders about using the Carbon Cycle Applet:
 - A) Set the land use, land sinks and ocean sinks lines to a horizontal position before you start your work
 - B) ***DO NOT PRESS RESET***...doing so would undo your horizontal line work.

Running the carbon Cycle Applet

<http://carboncycle.aos.wisc.edu/>

The Carbon Cycle Applet contains four input settings:
Fossil Fuels, Land Use Changes, Ocean Sinks and Land Sinks.

To begin the process of inputting your data you need to do the following:

- A) Set the Ocean Sinks, Land Sinks and Land Use graphs so that they become horizontal lines. Do this by sliding the open circles up or down on the green in for each of the inputs. We do this not because we actually expect these factors to remain unchanged into the future, but in order to isolate a single variable—Fossil Fuels—in our model. (You may have a hard getting them horizontal but do the best you can for this first run. For instance set Land Use to around +.8 at each time step. Ocean Uptake to -2.6, and Land Update to about -3.5)
- B) The fossil fuel graph is pre-set with the following carbon emission prediction values from the IPCC (Intergovernmental Panel on Climate Change) for the following future years:

(2020, 10.5 GT carbon per year)

(2040, 15.0 GT C/y)

(2060, 19.5 GT C/y)

(2080, 24.0 GT C/y)

(2100, 28.6 GT C/y)

The IPCC figures predict these enormous increases in carbon emissions because of projected population growth and the expected increase in people's standards of living into the future.

The shaded light blue area is the range the IPCC considers as potentially possible. The realistic expectation of the IPCC is the solid green line with open circles. Each open circle is slide able by you to set value your

Wedge Adjustments in 2020

Your job now is to reduce these carbon emissions values based on the **wedges you propose putting into place in 2020 and 2040**. For each wedge you put into place in 2020, you need to decrease the 2020, 2040, 2060, 2080 and 2100 carbon emissions values by one GT. For example, if you decided to implement 5 wedges in 2020, your new fossil fuel figures would be:

(2020, 5.5 GT carbon per year)

(2040, 10.0 GT C/y)

(2060, 14.5 GT C/y)

(2080, 19.0 GT C/y)

(2100, 25.3 GT C/y).

Wedge Adjustments in 2040

Next, you must repeat this process to account for the wedges you plan to put into place by 2040. For each wedge you put into place in 2040, you need to decrease the 2040, 2060, 2080 and 2100 carbon emissions values by one GT. For example, if you decided to implement 7 wedges in 2040, your new fossil fuel figures would be:

(2020, 5.5 GT carbon per year)

(2040, 3.0 GT C/y)

(2060, 7.5 GT C/y)

(2080, 12.8 GT C/y)

(2100, 18.3 GT C/y).

Run the Carbon Cycle Projection Simulation 1 (CO2 effect alone)

- C) Now that you have Steps A and B above completed you are ready to run the Carbon Cycle Applet to see if the wedges you put into place were enough to keep atmospheric CO2 levels below 450 ppm. When you click on the “**Run Projection**” button, keep an eye on the upper right-hand corner of the picture at the bottom of the screen where you will see the year and the atmospheric CO2 concentration appear. When the projection run is over, you can place your cursor on the green projection line for data on the CO2 value for each year of your projection. Write a brief statement about the results of your experiment including answers to the following questions and sketch the shape of the projection curve, locating the maximum, minimum and final values on your sketch.
- 1) What was the highest CO2 reading?
 - 2) In what year did it occur?
 - 3) What was the final reading in 2100?
 - 4) Was the CO2 curve increasing, decreasing or horizontal at the end?

Run the Carbon Cycle Projection Simulation 2 (add effect of land use and ocean)

- D) After running the Carbon Cycle Applet using the 12 wedges approach with all the other factors set to a horizontal position, run the Applet again—but this time use all of your background knowledge about the growth and decay of carbon sinks, your understanding of land use changes and everything you have learned about climate change solutions in order to make a more accurate prediction about what you believe the next 90 years have in store.

Imagine what you believe the appropriate Ocean Sinks levels will be in 2020, 2040, 2060, 2080 and 2100, then write a short statement about why you chose the levels you chose. Do the same for Land Sinks, Land Sources and Fossil Fuel sources. Record the levels you chose for each of these variables then run the Carbon Cycle Applet again and record the results of this second experiment including answers to the four questions above and a sketch of the final projection graph.